

# Non-universal dynamics of staggered non-equilibrium particle systems and Ising chains

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PACS numbers: 05.70.Ln, 75.10.Hk, 02.50.Ey

Published in J. Phys. A **31**, 541–549 (1998).

## Abstract

Non-universal dynamics is shown to occur in a one-dimensional non-equilibrium system of hard-core particles. The stochastic processes included are pair creation and annihilation (with rates  $\epsilon$  and  $\epsilon'$ ) and symmetric hopping rates which alternate from one bond to the next ( $p_A$ ,  $p_B$ ). A dynamical scaling relation between the relaxation time and the correlation length in the steady state is derived in a simple way for the case  $\epsilon' > p_A \gg p_B \gg \epsilon$ . We find that the dynamical exponent takes the non-universal value  $z = 2 \ln(\frac{\epsilon'}{\epsilon}) / \ln(\frac{p_B \epsilon'}{p_A \epsilon})$ .

For the special condition  $\epsilon + \epsilon' = p_A + p_B$ , where the stochastic system is in principle soluble by reduction to a free fermion system, the model is mapped to the Glauber dynamics of an Ising chain with alternating ferromagnetic bonds of values  $J_1$  and  $J_2$ , in contact with a quantum thermal bath. The full time dependence of the space-dependent magnetisation and of the equal time spin-spin correlation function are studied by writing the master equation for this system in the quantum Hamiltonian formalism. In particular we obtain the dispersion relations and rigorously confirm the results obtained for the correlation length and for the dynamical exponent.

# 1 Introduction

This paper discusses non-universal critical dynamics of non-equilibrium particle systems, in which dynamic exponents depend on microscopic parameters (ratios of transition rates).

Non-equilibrium particle systems with stochastic dynamics show properties ranging from steady state phase transitions to turbulence, shocks, and non-equilibrium analogues of strong fluctuations and critical behaviour [1, 2]. As with equilibrium critical phenomena, associated critical exponents are typically dependent on symmetries and dimensionalities and (for dynamics) conservation laws, but not on microscopic details. In particular the dynamic exponent  $z$  in many simple systems describes diffusive relaxation towards equilibrium ( $z = 2$ ), while in others it takes other parameter independent values, e.g.  $z = 3/2$  in the one-dimensional noisy KPZ or Burgers equations [3, 4, 5, 6].

Simple hard-core particle models of non-equilibrium stochastic dynamics can be mapped to quantum systems and in particular to spin models. This latter mapping is achieved either by a pseudo-spin representation of the particles or by associating particles with domain walls [7, 8, 9, 10, 11].

But it is well known that non-universal behaviour can exist in certain non-uniform spin systems with Glauber dynamics [12], such as that providing a simple model for freezing into non-equilibrium states and glassy dynamics [13, 14, 15, 16]. Here the relationship  $\tau \sim \xi^z$  between the relaxation time  $\tau$  and the equilibrium correlation length involves a parameter-dependent dynamic exponent  $z$  [17, 18, 19]. This suggests that analogous non-universal behaviour can exist in non-uniform stochastic particle systems, and this is demonstrated here.

The model considered is defined on a one-dimensional lattice with  $L$  sites. We divide this into two sub-lattices,  $A$  and  $B$ , containing even and odd sites respectively. Each site can be occupied by at most one particle. A pair of particles in adjacent sites can annihilate with rate  $\epsilon'$ . A pair can be created on two empty adjacent sites with rate  $\epsilon$ . A particle can hop to an empty nearest neighbour site with rate  $p_A$  if it is in sub-lattice  $A$  and with rate  $p_B$  if it is in sub-lattice  $B$ .

The correlation lengths characterising the steady state particle separations on each sub-lattice can be obtained by dynamic balance conditions within a mean field approximation. The characteristic time for diffusion across the smaller of the lengths,  $\xi$ , can then be estimated from the hopping process. This is

particularly simple to do when e.g. the annihilation rate is much greater than the creation rate (when  $\xi$  is large) and one hopping rate is much greater than the other. This simple procedure already provides the non-universal behaviour of the dynamic exponent  $z$ .

For a special case where the rates satisfy one constraint, the quantum spin Hamiltonian representing the particle dynamics reduces to a free fermion form after a Jordan-Wigner transformation [20]. The alternative and simpler way used here of achieving an exact solution for this case is by mapping the stochastic particle system to the Glauber dynamics of an Ising model with alternating bonds  $J_1, J_2$  [10, 21].

The correlation lengths and relaxation time readily provided by the simple argument in the particle picture thus translate, for the special rate relation, into results for the alternating bond Ising model, where they can be confirmed by exact calculation on the spin model.

The Glauber dynamics of this model is that resulting from coupling the static Ising system to a quantum thermal bath and considering the Van Hove limit of a weak interaction and very large times. By writing the master equation which describes the Glauber dynamics in a quantum Hamiltonian formalism, we can easily obtain the equation determining the time evolution of the equal-time spin-spin correlation function. From this quantity one can compute the equilibrium domain wall density in the Glauber problem which is equivalent to the steady state density of particles in the stochastic particle dynamics [10, 21]. By taking the Fourier transform of this equation we obtain a system of four linear equations which determine the dependence of the eigen-frequencies with the wave vectors of the two spin excitations. The solution of the associated secular equation gives us the dispersion relation. The behaviour of the frequency at low wave vectors determines the critical exponent  $z$ . We fully confirm the results obtained previously.

The structure of this paper is as follows: in section 2, we define the dynamics of the particle system in terms of the constituent stochastic processes (diffusion, annihilation and creation). Using the simple mean-field and random-walk arguments referred to above we obtain the relation between the relaxation time and the equilibrium correlation length and hence we determine the critical exponent  $z$ . In section 3 we present the Glauber dynamics of the alternating-bond Ising system, giving the rates of transition between the different configurations. We then go on to relate the spin flip processes to domain wall hopping, creation and annihilation, which translate to the particle processes. In section 4, we write down the master equation

in a quantum Hamiltonian formalism and derive the equation giving the time evolution of the spin-spin correlation function. Hence, we obtain the dispersion relation and from it we can again compute  $z$ . Finally, in section 5 we present our conclusions.

## 2 The reaction-diffusion system and its non-universal dynamics

The hard-core particle dynamics is as specified in the Introduction: a particle on the even sub-lattice may hop to its left nearest neighbour site, provided it is empty, or to its right nearest neighbour (if empty) at rate  $p_A$ ; the corresponding rate for hopping from the odd sub-lattice is  $p_B$ ; in addition, pairs of particles can appear at (or annihilate from) adjacent empty (or full) sites at rate  $\epsilon$  (or  $\epsilon'$ ). These processes are depicted in Table 1 (see below).

We now consider the description of the steady state. Here the particle densities  $\rho_A$  and  $\rho_B$  on the two sub-lattices will be uniform. Since the system is in equilibrium the processes of creation and annihilation have to balance each other. So one concludes that provided the mean field approximation applies,  $\epsilon' \rho_A \rho_B = \epsilon (1 - \rho_A)(1 - \rho_B)$ . The same must be true for the processes of diffusion. Hence  $p_A \rho_A (1 - \rho_B) = p_B (1 - \rho_A) \rho_B$ . One obtains from these relations  $\rho_A \sim \frac{x_A}{1+x_A}$ , where  $x_A = (\frac{\epsilon p_B}{\epsilon' p_A})^{1/2}$  and  $\rho_B \sim \frac{x_B}{1+x_B}$ , where  $x_B = (\frac{\epsilon p_A}{\epsilon' p_B})^{1/2}$ . These densities allow us to extract two distinct separation lengths, the average distances  $\rho_\alpha^{-1}$  between particles on each sub-lattice  $\alpha$ .

We will be particularly interested in the critical situation where these two characteristic lengths are both large. That occurs when  $\epsilon' \gg \epsilon$ , giving large particle separations, or where  $\epsilon \gg \epsilon'$  (large vacancies separations). For convenience we consider only the former case. It is also convenient to take  $p_A \gg p_B$ , to widely separate the two lengths. Both are still large provided  $\epsilon'/\epsilon \gg p_A/p_B$ . The shorter, controlling correlation length  $\xi$  is then

$$\xi \sim \rho_B^{-1} \sim x_B^{-1} = (\epsilon' p_B / \epsilon p_A)^{1/2}. \quad (1)$$

Now to determine the rate of approach to this steady state for  $\epsilon' \gg \epsilon$  one uses a modification, appropriate to the particle dynamics, of an argument given by [22]. For large times the evolution of the system is determined by the limiting relaxational process, namely particles diffusing until they meet, when they annihilate.

For situations close to the steady state, i.e. for long time critical dynamics (starting from generic non-equilibrium initial states), the characteristic time involved is that for particles to diffuse across the shorter steady state separation length,  $\xi$ , and to annihilate. Now to traverse two adjacent bonds both rates  $p_A$  and  $p_B$  enter and the effective diffusion rate is  $\frac{4p_A p_B}{p_A + p_B}$ . Thus the characteristic time is  $\tau \sim (\epsilon' p_B / \epsilon p_A) \frac{p_A + p_B}{4p_A p_B} \propto \xi^z$  giving

$$z = 2 \ln\left(\frac{\epsilon'}{\epsilon}\right) / \ln\left(\frac{\epsilon' p_B}{\epsilon p_A}\right) \quad (2)$$

in the regime of validity ( $\epsilon'/\epsilon \gg p_A/p_B \gg 1$ ). This is a non-universal result, being dependent on ratios of rates. The above reasoning is based on the mean field argument that we have used to determine  $\rho_A$  and  $\rho_B$ . Mean field theory determines correctly the characteristics of the steady state but it fails to predict the correct approach to the steady state [11]. This is due to the diffusion aspects of the problem, which are relevant in one dimension. The random-walk argument that we have given to determine  $\tau$  incorporates these aspects. We will see in the next sections that the same results are obtained by an exact calculation based on an equivalent Glauber-Ising dynamics. That equivalence applies when the rates satisfy

$$\epsilon + \epsilon' = p_A + p_B . \quad (3)$$

This relation is also sufficient to reduce the quantum Hamiltonian representing the particle dynamics to a free fermion form, which gives another possible method for solving the particle dynamics exactly.

Finally, we note that the exponent  $z$  changes continuously with the value of the rates of the diffusion and reaction processes, as given by equation (2). A similar example of a critical exponent which changes continuously was found in the study of the dynamics of the  $q$ -state Potts model [23]. In this system, the fraction of spins which never flip up to time  $t$ ,  $r(q, t)$ , decays like a power law  $r(q, t) \sim t^{-\theta(q)}$  when the initial condition is random. The exponent  $\theta(q)$  varies continuously with  $q$ .

### 3 An Ising system with generalised Glauber dynamics, and its relationship to reaction-diffusion processes

The Ising system considered is a spin  $\frac{1}{2}$  chain, where a spin on the even sub-lattice is coupled to its left nearest neighbour by a ferromagnetic Ising interaction of strength  $J_1$  and coupled to its right nearest neighbour by an

interaction of strength  $J_2$  ( $J_1 > J_2$ ) [17]. One can therefore write the Hamiltonian for this alternating-bond system in the form

$$H = - \sum_{l=1}^L J_l \sigma_l \sigma_{l+1} \quad (4)$$

where  $\sigma_l$  denotes the eigenvalues of  $\hat{\sigma}_l^z$ ,  $J_l = J_1$  for  $l$  odd and  $J_l = J_2$  for  $l$  even. The transitions between different configurations of the system, i.e. between different sets of values of the spins, are of the Glauber type, i.e. the system evolves by single spin flips [12]. The probability per unit time that a spin will flip from its value to the opposite one is taken to be [17]

$$\omega(\sigma_l) = \frac{\Gamma}{2} [1 - \frac{1}{2} \sigma_l (\gamma_l^+ \sigma_{l-1} + \gamma_l^- \sigma_{l+1})] \quad (5)$$

with

$$\gamma_l^\pm = \tanh(K_{l-1} + K_l) \pm \tanh(K_{l-1} - K_l) \quad (6)$$

where  $K_l = J_l/k_B T$ . Notice that if we take  $J_1 = J_2$  we will recover the usual Glauber rates. It can be easily verified that these rates satisfy detailed balance which is a sufficient condition for the steady state distribution of the Ising system to be a Gibbs distribution. This dynamics can be derived [24] by coupling the Ising system, described by Hamiltonian (4) to an ensemble of free fermion baths with a grand canonical probability distribution, i.e. with a density matrix given (in the case of no interaction) by  $\hat{\rho} = \exp[-\beta(\hat{H} - \mu\hat{N})]/Z$  where  $\hat{H}$  and  $\hat{N}$  are respectively, the Hamiltonian and the particle number operator of the free fermion bath,  $\beta$  is the inverse temperature and  $\mu$  the chemical potential. The quantity  $Z$  is the grand partition function and is just a normalisation factor. If the particle number of the fermion system is kept fixed, then the chemical potential is, for low enough temperatures, essentially equal to the Fermi energy of the system. The coupling is done via an interaction operator which couples the  $\hat{\sigma}^x$  component of each spin to a thermal bath whose probability distribution is the one given above. The different copies of the thermal bath are totally uncorrelated. If one takes the joint limit of the interaction strength  $\lambda$  going to zero and  $t \rightarrow \infty$  such that  $t\lambda^2$  is a constant (limit of Van-Hove) [25, 26] one obtains the transition rates given by (5), in the condition that the chemical potential (Fermi energy) of the fermion bath is much larger than the couplings  $J_1$  and  $J_2$ . If that is not the case then the transition rates still have the form (5), but the parameter  $\Gamma$  is no longer

Initial State	Final State	Transition Rate
$\uparrow \circ \uparrow \circ \uparrow$	$\uparrow \bullet \downarrow \bullet \uparrow$	$\frac{\Gamma}{2} [1 - \tanh(K_1 + K_2)] = \epsilon$
$\uparrow \bullet \downarrow \bullet \uparrow$	$\uparrow \circ \uparrow \circ \uparrow$	$\frac{\Gamma}{2} [1 + \tanh(K_1 + K_2)] = \epsilon'$
$\uparrow \circ \uparrow \bullet \downarrow$	$\uparrow \bullet \downarrow \circ \downarrow$	$\frac{\Gamma}{2} [1 + \tanh(K_1 - K_2)] = p_A$
$\uparrow \bullet \downarrow \circ \downarrow$	$\uparrow \circ \uparrow \bullet \downarrow$	$\frac{\Gamma}{2} [1 - \tanh(K_1 - K_2)] = p_B$

Table 1: Processes of transition for a spin in the even sub-lattice and the equivalent particle processes. The spin which is to flip is the central one. The coupling strength of the left bond is  $J_1$ , and of right bond  $J_2$ . An empty circle in the dual lattice is to be identified with a vacancy and a full circle with the presence of a particle. For a spin in the odd sub-lattice the rates of the last two processes should be interchanged.

a constant and depends on the value of the neighbouring spins. We will consider that we can take  $\Gamma$  as a constant.

It is well known that there is a duality relation between Glauber dynamics and the reaction-diffusion model of hard-core particles discussed above [10, 21]. To see this we take a given configuration of the Ising system and consider the lattice of sites located in the middle of the bonds between the Ising spins (dual lattice). If the neighbouring Ising spins have different signs then we place a particle at that site of the dual lattice. Otherwise we leave the site empty. In this way we map domain walls in the Ising system to particles in the dual lattice. It can be shown that this mapping has a precise mathematical meaning [27]. The different possible processes of transition for a given spin, its translation in terms of particle processes, the rates associated and their relation are given in table 1. Notice that the sum of the rates of the first two processes is equal to the sum of the last two in agreement with (3). This shows that this system is in the class of systems that are integrable (the precise meaning of this word will be made clear below) through free fermions [7]. This allows the computation of correlation functions for a set of distributions of initial configurations [11, 27]. Here we will not pursue this point. If we now take the limit of low temperatures ( $K_1 \gg K_2 \gg 1$ ) we see that the rates of pair annihilation ( $\epsilon'$ ) and of diffusion from the strong bond to the weak bond  $p_A$  are equal to  $\Gamma$ . On the other hand the rate of diffusion from a weak bond to a strong bond is

$p_B \sim \Gamma \exp(-2(K_1 - K_2))$ . Finally the rate of pair creation is  $\epsilon \sim \Gamma \exp(-2(K_1 + K_2))$ . So we see that these two last processes are exponentially suppressed at low temperatures. Nevertheless they have to be taken into account for a proper description of the steady state and diffusive dynamics, as we saw in section 2. We will see in the next section that the results given in section 2 may be obtained by an exact calculation.

## 4 The master equation in the quantum Hamiltonian formalism

In this section we will study the time evolution of the spin-spin correlation function. For that we need the full master equation for the evolution of the Ising system with rates given by (5). Since the master equation is a linear equation a particularly convenient way to write it is to use an operator formalism which assigns to each configuration of Ising spins a vector  $|\underline{n}\rangle$  in an Hilbert space. The probability distribution for the different configurations at a given time  $t$  can then be written as a state-vector  $|\Psi_t\rangle = \sum_{\underline{n}} P(\underline{n}, t) |\underline{n}\rangle$  [28], where  $P(\underline{n}, t)$  is the probability to find configuration  $\underline{n}$  at time  $t$  and is a solution of the master equation. The set of different  $|\underline{n}\rangle$  is supposed to be orthonormal and complete. One can then write the master equation in the compact form  $\partial_t |\Psi_t\rangle = -\hat{T} |\Psi_t\rangle$ , where  $\hat{T}$  is a linear and in general non-hermitian operator that for two state systems with local interactions can be written as a quantum spin Hamiltonian. The average values of quantities like the Ising spins can also be conveniently represented in this language as  $\langle \sigma_j(t) \rangle = \langle s | \hat{\sigma}_j^z | \Psi_t \rangle$ , where  $\hat{\sigma}_j^z$  is the Pauli spin matrix at site  $j$  and  $\langle s | = \sum_{\underline{n}} \langle \underline{n} |$ . Substituting the state vectors and the operators by their definitions and using the orthonormality relations one sees that one obtains the usual definition of average over configurations. One also can, given the correspondence with the Schrödinger equation, define an Heisenberg representation of the operators by  $\hat{A}(t) = e^{\hat{T}t} \hat{A} e^{-\hat{T}t}$ , where  $\hat{A}$  is a generic operator. These operators obey the equation of motion  $\frac{d\hat{A}}{dt} = [\hat{T}, \hat{A}(t)]$ . It should be stressed that this is only a convenient way to represent the master equation for this system and is not related to the intrinsic quantum dynamics of the Ising system which is given by the Hamiltonian (4). In fact, the dynamics is in this case generated by the interaction of the Ising spins with the thermal bath and cannot be deduced from the form of (4) alone. For the system that we have studied in the previous section the operator  $\hat{T}$  has

the form

$$\hat{T} = \frac{\Gamma}{2} \sum_{l=1}^L (1 - \hat{\sigma}_l^x) [1 - \frac{1}{2} \hat{\sigma}_l^z (\gamma_l^+ \hat{\sigma}_{l-1}^z + \gamma_l^- \hat{\sigma}_{l+1}^z)] \quad (7)$$

as can be seen if we consider the matrix elements  $\langle \underline{n}' | \hat{T} | \underline{n} \rangle$  and  $\langle \underline{n} | \hat{T} | \underline{n} \rangle$ , where  $\underline{n}'$  is a configuration differing from  $\underline{n}$  by the flip of a single spin  $\sigma_l$ , say. We obtain

$$\begin{aligned} \langle \underline{n}' | \hat{T} | \underline{n} \rangle &= -\frac{\Gamma}{2} [1 - \frac{1}{2} \sigma_l (\gamma_{l-1}^+ \sigma_{l-1} + \gamma_{l+1}^- \sigma_{l+1})] \\ \langle \underline{n} | \hat{T} | \underline{n} \rangle &= \frac{\Gamma}{2} \sum_{l=1}^L [1 - \frac{1}{2} \sigma_l (\gamma_{l-1}^+ \sigma_{l-1} + \gamma_{l+1}^- \sigma_{l+1})] \end{aligned} \quad (8)$$

where  $\sigma_l$ , etc., are the values of the Ising spins in configuration  $\underline{n}$ . The first relation gives (up to a minus sign) the transition rate between configurations  $\underline{n}$  and  $\underline{n}'$ . The second relation gives the total rate of transition out of configuration  $\underline{n}$ . This agrees with the general definition of the  $\hat{T}$  operator [29]. The form (7) and associated equations of motion are particularly convenient for they allow the calculation of multiple time spin-spin correlation functions. In the same way one can represent the master equation for the particle dynamics in terms of a  $\hat{T}$  operator which is also a quantum spin Hamiltonian. The duality relation between these two models can be given a precise mathematical meaning by means of a similarity transformation between the operators of the two models [27]. In particular, the  $\hat{T}$  operator of the generalised Glauber dynamics maps to the  $\hat{T}$  operator of the particle system. This later operator can be expressed in terms of free fermions by a Jordan-Wigner transformation [20] and therefore the system is completely integrable. Many correlations functions relevant for the study of the particle dynamics and the associate Glauber problem can hence be obtained.

If one now considers the Heisenberg equation of motion for  $\hat{\sigma}_j^z(t)$  and takes the average value on some arbitrary initial state  $| \Psi \rangle$  in the way indicated above one obtains the differential equation giving the time development of the average space dependent magnetisation [17]. This equation can be solved by defining the Fourier transforms  $m_k^+$  and  $m_k^-$  of the magnetisation at even and odd sites respectively, with the wave vector  $k$  equal to  $k = \frac{2\pi}{L}n$  with  $n$  an integer and such that  $k$  lies in the limits  $-\frac{\pi}{2} \leq k < \frac{\pi}{2}$  (we have chosen the lattice spacing to be one). Notice that the Brillouin zone has been reduced to half of the size it would have if  $J_1 = J_2$  since the lattice has now a periodicity of two. This procedure gives us a system of two equations

coupling  $m_k^+$  and  $m_k^-$  for every  $k$ . The relaxational eigenvalues of this system are

$$\omega_k^\pm = 1 \pm [\cos^2 k \tanh^2(K_1 + K_2) + \sin^2 k \tanh^2(K_1 - K_2)]^{1/2} \quad (9)$$

where  $\omega_k^\pm$  stands for the lower (acoustical) and upper (optical) branches of this dispersion relation. The late (critical) dynamics is determined by the low  $k$  modes of the acoustical branch  $\omega_k^-$ . The characteristic length associated with the decay of the local magnetisation in the steady state can be found from the value of  $|k|$  ( $= \frac{2\pi}{\xi}$ ) making the right hand side of (9) vanish. We therefore take the analytic continuation of (9) to the complex plane. We obtain the space decaying mode with infinite lifetime (which decays over a characteristic length  $\xi$ ). For low temperatures  $\xi$  is given by

$$\xi \sim \exp(2K_2) . \quad (10)$$

Therefore at low  $k$  and low temperatures one can write the dispersion relation as

$$\omega_k^- = 2\Gamma\xi^{-z} (1 + (k\xi)^2) \quad (11)$$

with  $\xi$  given by (10) and

$$z = 2 + \frac{J_1 - J_2}{J_2} \quad (12)$$

a result given in [17]. Now to determine the time evolution of the density of particles we need to compute the equivalent quantity in the Glauber language. This quantity is the domain wall local density [10] which is given by  $\frac{1}{2}(1 - \langle \sigma_l(t)\sigma_{l+1}(t) \rangle)$ . We therefore need the equation giving the time evolution of the spin-spin correlation function. Notice that these correlations do not involve only eigenmodes of (7) orthogonal to the slowly decaying ones, so their relaxation follows closely the slowest processes in the system. Using the transfer matrix formulation we can write  $\frac{d}{dt}\langle s | \hat{\sigma}_l^z(t)\hat{\sigma}_m^z(t) | \Psi \rangle = \langle s | [\hat{T}, \hat{\sigma}_l^z(t)\hat{\sigma}_m^z(t)] | \Psi \rangle$  where  $|\Psi\rangle$  is as above a general initial state. Substituting  $\hat{T}$  by its expression given in (7) we obtain the equations

$$\begin{aligned} \frac{d}{dt}\langle \sigma_l(t)\sigma_m(t) \rangle &= -2\Gamma\langle \sigma_l(t)\sigma_m(t) \rangle + \frac{\Gamma}{2}[\gamma_l^+\langle \sigma_{l-1}(t)\sigma_m(t) \rangle + \gamma_l^-\langle \sigma_{l+1}(t)\sigma_m(t) \rangle \\ &\quad + \gamma_m^+\langle \sigma_l(t)\sigma_{m-1}(t) \rangle + \gamma_m^-\langle \sigma_l(t)\sigma_{m+1}(t) \rangle] \end{aligned} \quad (13)$$

if  $l \neq m$  and the boundary condition  $\langle \sigma_l(t)\sigma_l(t) \rangle = 1$ . Since these equations are linear this boundary condition may be ignored. It can be later enforced by superposing different solutions [12]. To solve these

equations we introduce Fourier transforms of the quantities  $\langle \sigma_l(t)\sigma_m(t) \rangle$  when  $l$  and  $m$  are even,  $l$  is even and  $m$  is odd, etc. The two wave vectors  $k$  and  $k'$  which now appear are defined as above. We obtain a system of four linear equations. The associated secular equation has four solutions corresponding to having two excitations in the acoustic branch, one in the acoustic and one in the optical branch (two solutions) and two excitations in the optical branch. The frequency  $\omega_{kk'}$  is given by

$$\omega_{kk'} = \omega_k + \omega_{k'} \quad (14)$$

where  $\omega_k$  and  $\omega_{k'}$  are given by one of the solutions in (9). If we now take the translational average of the correlation function  $\langle \sigma_l(t)\sigma_{l+1}(t) \rangle$  we see that only the terms for which  $k' = -k$  remain in the Fourier sum. Hence the decay of density of domain walls (density of particles) is determined by the acoustical branch of  $\omega_{k,-k}$  for low  $k$ . At low temperatures we just obtain  $\omega_{k,-k} = 2\omega_k^-$  with  $\omega_k^-$  given (11). The interparticle distance in the steady state is given by the value of  $k$  that makes  $\omega_{k,-k}$  vanish. This coincides with (10). The relaxation time  $\tau$  is given by  $\tau = \xi^z / 4\Gamma$  ( $\tau^{-1} = \omega_{0,0}$ ). This translates into (2) using the relationships between rates in the Ising and particle pictures (see Table 1) in the low temperature limit. So we recover the results obtained in section 2 in a rigorous way. It can also be seen from (14) that dynamical scaling still holds for this model. This is nontrivial, since as pointed out above there are two correlation lengths in the problem.

## 5 Conclusions

We have studied a generalised reaction-diffusion process, and the related model of generalized Glauber dynamics for a staggered Ising model. We obtained the following results

- i) The dynamical exponent  $z$  relating the relaxation time to the equilibrium correlation length is non-universal, depending on the ratios of rates or the ratio between the couplings of the Ising model.
- ii) We found that dynamical scaling still holds, i.e. the dispersion relation still depends on a function of a single parameter  $k\xi$  where  $k$  is the wave-vector and  $\xi$  is the equilibrium correlation length.

It would be interesting to investigate the exact solution of this problem from the point of view of the particle dynamics, since one can diagonalize the dynamical operator  $\hat{T}$  by using free fermions. Also it would

be interesting to study if there is any change in the dynamical exponent  $z$  when one goes away from the free fermion condition. One would expect, based on the general argument given in section 2 that the relationship of  $z$  to the basic rates does not change provided that  $\epsilon'/\epsilon \gg p_A/p_B \gg 1$ . This means that in the long-time limit the system has a very low density of particles and therefore one should expect that the interactions are unimportant. Preliminary simulations seem to confirm this result [30].

**Acknowledgments:** It is a pleasure to acknowledge many helpful discussions with Gunter Schütz. We would also like to thank the International Center of Theoretical Physics for kind hospitality during the early stages of this work. R.B.S. would like to thank Rutgers University for kind hospitality during the later stages of this work. J. E. S. is supported by the Grant: PRAXIS XXI/BD/3733/94 - JNICT - PORTUGAL.

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